A New Modified Sharpe Ratio Method for Ranking Funds with Negative Excess Returns

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Abstract

The Sharpe (1966) ratio is the canonical risk-adjusted performance measure in investment management, yet it fails to produce economically meaningful rankings when portfolio excess returns turn negative—precisely the situation that characterized global equity markets in 2022. When all funds in a peer group generate negative excess returns, a fund with higher volatility and weaker performance can paradoxically display a higher (less negative) Sharpe ratio, inverting the intended risk-return ordering. A second, related limitation arises when return distributions depart from normality, rendering variance-based measures unreliable for fund comparison. The R-Sharpe ratio (RSR), a novel risk-adjusted performance measure, is introduced to resolve the negative-excess-return ranking anomaly by adding a peer-group-minimum constant (Δs) to the excess return numerator. This shift places all ratios in positive territory while preserving the mean-variance interpretation of the original Sharpe ratio. The RSR is benchmarked against Israelsen's (2005) modified Sharpe ratio and three downside-risk measures—the Modified Value-at-Risk (MVaR) Sharpe ratio, Lower Partial Moments (LPM), and the

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Omega ratio—using daily returns of 26 globally diversified equity funds over the bear market period September 2021–September 2022. The study demonstrates that the choice of performance measure materially influences fund rankings and investor assessments, with significant practical implications for fund selectors operating in negative market environments.

Keywords: Sharpe ratio · R-Sharpe ratio · Modified Sharpe ratio · Value at Risk · Lower Partial Moments · Downside Risk · Negative Excess Return

Non-Technical Summary

The Sharpe ratio, introduced by William Sharpe in 1966, is the most widely used measure for evaluating investment fund performance relative to risk. It divides a fund's excess return over the risk-free rate by its return volatility. Despite its universal adoption, the Sharpe ratio has a critical structural flaw that becomes apparent precisely when it matters most: during sustained market downturns. When all funds in a peer group generate negative excess returns—as happened in global equity markets in 2022, when the MSCI World Index fell approximately 18%—the Sharpe ratio systematically inverts fund rankings. A fund with worse performance and higher volatility can display a higher (less negative) ratio than a better-performing fund, effectively rewarding risk rather than return. This paper identifies and addresses this flaw.

The R-Sharpe ratio: This paper introduces the R-Sharpe ratio (RSR) as a direct and tractable correction for the negative-return anomaly. The RSR adds a peer-group minimum constant (Δs)—the absolute value of the most negative excess return in the peer group—to each fund's excess return numerator. This shift places all ratios in positive territory without distorting the relative risk-return ordering, and preserves the economic logic of Markowitz's (1952) mean-variance framework. Applied to 26 globally diversified equity funds from the GELD-Magazin peer group "Equity fund of funds global – dynamic" over the bear market period September 2021 to September 2022, the RSR produces fund rankings that are fully consistent with investors' actual performance preferences.

Alternative benchmarks: The RSR is benchmarked against Israelsen's (2005) modified Sharpe ratio and three downside-risk measures: the Modified Value-at-Risk (MVaR) Sharpe ratio, Lower Partial Moments (LPM), and the Omega ratio.

Using GARCH-NIG-filtered return distributions for tail-risk estimation and Jarque-Bera normality tests, the following findings emerge: (i) the classical Sharpe ratio produces counterintuitive, economically inconsistent fund rankings when all excess returns are negative; (ii) the new R-Sharpe ratio yields rankings fully consistent with the Markowitz (1952) mean-variance framework; (iii) seven of 26 funds exhibit statistically significant non-normal return distributions; and (iv) MVaR-based rankings reveal asymmetric tail risk that variance-based measures overlook. The results demonstrate that performance measure selection is non-trivial and materially affects fund ranking outcomes during market stress periods.

Practical implications: The choice of performance measure is not a technical detail—it materially determines which funds are identified as top performers during market downturns. The findings have direct relevance for fund selectors, portfolio managers, and institutional investors who evaluate fund performance in challenging market environments. The R-Sharpe ratio offers a simple, theoretically grounded, and practically implementable improvement that preserves the familiarity of the Sharpe ratio while eliminating its ranking failures under negative excess returns.

Contents

1. Introduction

The performance evaluation of investment funds is a cornerstone of modern portfolio management, yet the methodology faces acute challenges during sustained market downturns. The year 2022 delivered the sharpest global equity market decline since the 2008 financial crisis, with the MSCI World Index falling approximately 18% in U.S. dollar terms, exposing a fundamental flaw in conventional performance measurement. Markowitz's mean-variance model indicates that investors should select the optimal risky portfolio with the highest Sharpe ratio within a feasible set. According to Van and Duong (2021), the Sharpe ratio is a natural definition of a performance measure under this framework. However, the mean-variance model assumes negative exponential utility and normally distributed asset returns—assumptions that are routinely violated in practice, and particularly so during market stress episodes.

Prior literature documents that the Sharpe ratio (SR) generates systematic biases in performance evaluation: (1) when excess returns are negative, and (2) when return distributions deviate from normality—since SR is derived under the mean-variance model with the strict assumption of either quadratic preferences or normally distributed returns (Cerny, 2003). Dowd (1999) emphasized the importance of net portfolio exposures and proposed a Value-at-Risk approach to risk-return analysis; Dowd (2000) subsequently criticized the normality assumption underlying the Sharpe ratio's risk measure. Harding (2002) raised a further fundamental concern: since any positive return contributes to variance, removing the highest returns from a portfolio can paradoxically increase the Sharpe ratio by reducing "risk"—a reductio ad absurdum. Lo (2002) demonstrated that the Sharpe ratio's constituents are estimated quantities subject to statistical error, while Goetzmann et al. (2002) showed that option-like strategies can systematically inflate reported Sharpe ratios, exposing the measure to manipulation. When returns are negative, the SR produces counterintuitive rankings because a larger negative numerator divided by a smaller denominator yields a less negative ratio, effectively rewarding volatility over performance. Prior refinements include Israelsen's modified Sharpe ratio, which adjusts the denominator exponent when excess returns are negative. Yuan and Yuan (2023) provide recent empirical evidence confirming that performance measure selection systematically alters fund rankings, particularly during periods of market stress. For distributional concerns, downside risk measures—including Modified Value-at-Risk (Favre & Galeano, 2002), Lower Partial Moments, and the Omega ratio (Keating & Shadwick, 2002a, 2002b)—provide richer characterizations of tail risk. Therefore, this study examines which performance measurement approaches are most reliable for negative returns and non-normal return distributions.

Daily returns of 26 equity funds from September 2021 to September 2022 are analyzed. The classical Sharpe ratio is employed alongside two refined measures – Israelsen's modified Sharpe ratio and the newly introduced R-Sharpe ratio – to evaluate fund performance during the 2022 bear market. In addition, three modern downside risk measures (MVaR Sharpe ratio, Lower Partial Moments, and Omega ratio) are applied to assess non-normal return characteristics. The downside risk of an investment is the maximum loss that can occur owing to uncertainty in realized returns (Dowd, 2005). Investors need performance measures that can provide information on downside risk potential while also capturing upside return characteristics. GARCH-NIG-filtered residuals are used for MVaR estimation. All three downside estimators incorporate the first four moments of the return distribution in their computation.

The remainder of this paper is organized as follows. Section 2 reviews the two methodologies of the refined Sharpe ratios and presents three approaches for calculating nonparametric estimations and downside risks. The data basis and research methodology are described in Section 3. Section 4 presents the empirical results of the study. The final section is the conclusion.

2. Approaches to measuring performance

2.1 Approaches to measuring performance with negative returns

Several alternative refinements of the original Sharpe ratio have been developed to rank funds in periods of negative excess returns. Modified versions of the Sharpe ratio are found to generally result in more consistent performance rankings than the original Sharpe ratio.

2.1.1 Performance measures based on volatility

The normal distribution is based on the assumption that all returns are symmetrically distributed around their mean. It is a theoretical model and is fully defined by the parameters "expected value" and "variance". The normal distribution is a good approximation for the long-term returns of the global bond and stock market.

2.1.2 Israelsen's modified Sharpe ratio

Many studies have proposed adjustments to the traditional Sharpe Ratio to overcome two major shortcomings, namely the inability to account for negative returns and higher moments. For example, Israelsen's study suggests adding an exponent to the standard deviation (risk denominator) to improve the Sharpe ratio estimate when excess returns ($r_p - r_f$) are negative.

Israelsen's modified Sharpe ratio (ImSRp) is calculated using historical returns as follows:

$$ImSR_p = \frac{r_p - r_f}{\sigma_p \left[\frac{ER}{abs\ ER}\right]}$$

with:

ER : excess return portfolio p

r_f : arithmetic mean of the risk-free interest rate

r_p : arithmetic mean of the portfolio return p

σ_p : empirical standard deviation of the portfolio return

2.1.3 R-Sharpe ratio

The author's R-Sharpe ratio method is based on the idea of adjusting risk premiums in such a way that portfolios or funds with negative returns can also be correctly ranked according to their expected return/variance ratio.

The R-Sharpe ratio indicates the risk premium adjusted by $delta_{(s)}$ per unit of total risk assumed. Here, $delta_{(s)}$ = delta (r_{pw};1) = s2 - s1, where r_{pw} is the arithmetic mean of the returns of the worst performance portfolio in the peer group.

The R-Sharpe ratio (RSR_p) is calculated using historical returns as follows:

$$RSR_p = \frac{ER + \Delta_{(s)} + r_f}{\sigma_p}$$

with:

ER : excess return portfolio p

$\Delta_{(s)}$: difference delta (r_{pw};1) = s2 – s1

r_f : arithmetic mean of the risk-free interest rate

r_{pw} : arithmetic mean of the return of the weakest performing portfolio in the peer group

σ_p : empirical standard deviation of the portfolio return

The general rule is that the higher the R-Sharpe ratio, the better the return-to-risk performance of a particular fund. The use of this method is reasonable if funds produce negative return-to-risk ratios. The R-Sharpe ratio is quite simple to calculate and gives logically comprehensible results. Cross-asset class comparisons are possible if the funds to be compared have similar distribution characteristics.

2.2 Downside performance measure

If the returns are not normally distributed, for example if funds use options for leverage or hedging, the risks associated with fat tails or left-skewed distributions are significantly underestimated and the performance is presented too positively (Favre and Singer, 2002). The use of derivatives in a portfolio can cause a positive or negative skewness in the return distribution and at the same time have an impact on the kurtosis of the return distribution. A change in the skewness and kurtosis of the return distribution in turn has the effect that the probability of achieving a certain return changes and the probability assumptions of the normal distribution no longer fully apply. This is the reason why Favre and Galeano (2002) include skewness and kurtosis in the calculation by deriving the modified Value at Risk based on the Cornish-Fisher development. Furthermore, according to Chen, He & Zhang (2011), the Omega Ratio is another useful ratio in the context of non-normally distributed returns and downside risks.

2.2.1 Modified (MVaR) Sharpe ratio

According to Gregoriou & Gueyie (2003), the modified Sharpe ratio considers the excess return of an investment to the ratio of a modified Value at Risks, whereby this ratio extends the normal Sharpe ratio by the factors skewness and kurtosis.

According to Gregoriou et al. (2003), the formula can be defined mathematically as follows:

$$MSR_p = \frac{r_p - r_f}{MVAR_p}$$

$$MVAR_p = \mu - \{z_c + \frac{1}{6} * (z_c^2 - 1) * S + \frac{1}{24} * (z_c^3 - 3z_c) * K - \frac{1}{36} * (2z_c^3 - 5z_c) * S^2\} * \sigma$$

with:

μ : arithmetic average of returns

r_f : arithmetic mean of the risk-free interest rate

r_p : arithmetic mean of the portfolio return p

σ_p : empirical standard deviation of the portfolio return

z_c : z-score of the normal distribution (1-alpha)

S : skewness

K : kurtosis

2.2.2 Lower Partial Moments

The lower partial moments (LPMs) measure risk by negative deviations of the stock returns in relation to a minimal acceptable return, or return threshold τ. As LPMs consider only negative deviations of returns from τ, they seem to be a more appropriate measure of risk than the standard deviation, which considers both negative and positive deviations from the expected return. The advantage of the LPM measures is that no prerequisites with regard to the form of distribution have to be fulfilled. The LPM measures can be applied to any distribution of returns.

The literature proposes a minimum return τ of 0 percent as nominal capital preservation, the inflation rate as real capital preservation, the risk-free interest rate as achieving the minimum opportunity cost, the expected value of the return on a given asset as ensuring the expected increase in wealth, and a market index as comparing performance with the market.

Given a sample in the size of T observations, the following distribution-free estimator of the nth-order LPM measure is obtained:

$$LPM_{n,p}(\tau) = \frac{1}{T}\sum_{t=1}^{T} max\left[\tau - r_{t,p}, 0\right]^{n}$$

About the parameter n, LPM with the order $0 < n < 1$ can express 'risk seeking', for $n = 1$ 'risk neutrality', and $n > 1$ 'risk aversion' behaviour for the investor. Thus, the higher n the more risk averse an investor. The LPMs of order 0 and 1 are, respectively, the shortfall probability and the expected shortfall.

2.2.3 Omega ratio

In a seminal contribution, Keating and Shadwick (2002a) and Keating and Shadwick (2002b) introduced the Omega ratio, and claimed that this universal performance measure, designed to redress the information impoverishment of traditional mean-variance analysis would address these concerns. They emphasize that the Omega metric has the great advantage over traditional measures to encapsulate all information about risk and return as it depends on the full return distribution, as well as to avoid looking at a specific level as this measure is provided for all levels, hence allowing each investor to assess their own risk appetite.

Strictly speaking, the omega ratio is defined as the probability-weighted ratio of gains over losses at a given level of expected return. In financial words, this ratio determines the quality of the investment bet relative to the return threshold.

Omega (Ω) is calculated by adjusting the excess return with the LPM measures of order 1, 2 or 3.

According to Bacon (2008), the Omega Sharpe ratio is simply Ω - 1, which should produce identical performance rankings to the original Omega (Ω) ratio, which can be represented as follows (Keating and Shadwick, 2002):

$$OSR_p = \frac{r_p - r_f}{Downside\ potential}$$

$$Downside\ potential = \sum_{i=1}^{i=n} max\ (r_\tau - r_p, 0)$$

3. Data and Research Methodology

Daily returns are calculated based on the daily redemption values of the equity mutual funds.

For the performance analysis, the period from September 2021 to September 2022 is used. The choice of the one-year interval is justified by the existence of negative excess returns as a result of the negative stock market phase in 2022.

Another argument for this time horizon is the possibility of comparing the risk-adjusted and downside risk results after applying different performance measures with the peer group of GELD-Magazin, which ranks funds by return.

A threshold return of 10% and a risk-free rate of 0.50% are applied. Previous analyses show that the choice of interest rate has little or no impact on the results. Although level differences may arise due to the choice of interest rate, they do not exert any influence on the ranking (Roßbach, 1991).

Daily returns of 26 equity funds from September 2021 to September 2022 are collected. These funds represent the peer group “Equity fund of funds global – dynamic” of the GELD-Magazin, No. 11/2022.

4. Empirical Results

4.1 Results of approaches to measuring performance with negative returns

4.1.1 Ranking based on returns

Table 1 reports the first two moments of the peer group. As shown in the Table 1, all funds exhibit negative returns. The fund with the lowest volatility (12.74%) has a higher return compared to the fund with the highest volatility (22.28%).

Table 1: Peer group ranked by returns

ISIN	Equity Funds	Return %	Volatility %	Ranking
AT0000707682	Erste Best of World	-6.55	15.25	1
AT0000722988	FWU TOP Trends	-6.79	13.64	2
AT0000A2HTY1	R&B Aktien Global Aktiv	-7.23	12.74	3
AT0000754700	Hypo Dynamic Equity	-7.44	13.87	4
AT0000A1VNX9	Faktorstrategie Akt. Global	-7.51	17.72	5
IE00B537M394	GAM Star Comp. Glob. Eq.	-8.32	16.07	6
AT0000703491	Piz Buin Global	-9.36	15.05	7
AT0000A2B6F7	Hypo Vlbg. Weltportf. Akt.	-9.55	14.64	8
AT0000746581	All Trends	-9.81	14.02	9
DE0005314447	UniStrategie: Offensiv	-11.39	14.09	10
AT0000708771	Apollo Dynamisch	-12.40	13.83	11
AT0000784889	3 Banken Werte Growth	-12.53	16.76	12
AT0000A2GYP1	Nachhaltigkeitsstrat. Akt.	-12.55	14.98	13
AT0000801170	All World	-12.57	13.90	14
DE0005326789	UniSelection: Global	-12.64	13.14	15
AT0000811427	Equity s Best-Invest	-12.72	13.82	16
AT0000746524	FWU TOP spekulativ	-13.70	14.71	17
LU0126855997	WWK Select Chance	-14.01	14.71	18
AT0000746516	FWU TOP offensiv	-14.18	15.10	19
AT0000719893	Pro Invest Aktiv	-15.53	15.63	20
AT0000731666	Portf. Next Generat. ZKB Oe	-17.05	17.20	21
AT0000784830	3 Banken Aktienf.-Selektion	-17.13	13.87	22
AT0000817945	Aktienstrategie global	-19.63	15.10	23
LU0073129206	UBS (Lux) Strategy Fund Eq.	-21.83	16.73	24
LU2123743341	BlackRock Mult.-Theme Eq.	-24.22	22.28	25
LU2001994727	UBAM Multif. Seul. Trends	-27.18	17.33	26

Notes: This was the ranking published by GELD-Magazine in its November 2022 issue, which was used to determine the winner of the Austrian Fund of Funds Award. Due to the negative returns, GELD-Magazin decided not to factor in risk as it normally would when compiling the rankings. The official winner of the award was the "Erste Best of World" Equity Fund, despite higher volatility than the next three funds in the ranking. GELD-Magazine had already announced this approach regarding negative returns in its terms and conditions prior to compiling the ranking.

4.1.2 Rankings by Application of the Sharpe Ratio

Table 2 lists the ranking based on the Sharpe ratio. As a result of the negative stock market performance in 2022, all funds have a negative Sharpe ratio. In such cases it is often considered paradoxical that a fund with greater standard deviation and weaker performance may nonetheless have a higher (less negative) excess return Sharpe ratio and thus be considered to have been "better". This effect is also observed in the present study. Table 2 illustrates that the fund in position 1 has, despite its higher volatility and lower return, the better SR compared with the fund in position 2. Thus, it is suggested that, for performance evaluation, the SR in phases of negative returns should not be used.

Table 2: Peer group with Sharpe ratio (SR) results

ISIN	Equity Funds	Return %	Volatility %	SR*	Ranking
AT0000A1VNX9	Faktorstrat. Akt. Glob.	-7.51	17.72	-0.42	1
AT0000707682	Erste Best of World	-6.55	15.25	-0.43	2
AT0000722988	FWU TOP Trends	-6.79	13.64	-0.50	3
IE00B537M394	GAM Star Comp. Glob.	-8.32	16.07	-0.52	4
AT0000754700	Hypo Dynamic Equity	-7.44	13.87	-0.54	5
AT0000A2HTY1	R&B Aktien Global Ak-	-7.23	12.74	-0.57	6
AT0000703491	Piz Buin Global	-9.36	15.05	-0.62	7
AT0000A2B6F7	Hypo Vlbg. Weltportf.	-9.55	14.64	-0.65	8
AT0000746581	All Trends	-9.81	14.02	-0.70	9
AT0000784889	3 Banken Werte Growth	-12.53	16.76	-0.75	10
DE0005314447	UniStrategie: Offensiv	-11.39	14.09	-0.81	11
AT0000A2GYP1	Nachhaltigkeitsstrat.Akt.	-12.55	14.98	-0.84	12
AT0000708771	Apollo Dynamisch	-12.40	13.83	-0.90	13
AT0000801170	All World	-12.57	13.90	-0.90	14
AT0000811427	Equity s Best-Invest	-12.72	13.82	-0.92	15
AT0000746524	FWU TOP spekulativ	-13.70	14.71	-0.93	16
AT0000746516	FWU TOP offensiv	-14.18	15.10	-0.94	17
LU0126855997	WWK Select Chance	-14.01	14.71	-0.95	18
DE0005326789	UniSelection: Global	-12.64	13.14	-0.96	19
AT0000731666	Portf. Next Generat.	-17.05	17.20	-0.99	20
AT0000719893	Pro Invest Aktiv	-15.53	15.63	-0.99	21
LU2123743341	BlackRock Mult.-Theme	-24.22	22.28	-1.09	22
AT0000784830	3 Banken Aktienf.-Selek-	-17.13	13.87	-1.23	23
AT0000817945	Aktienstrategie global	-19.63	15.10	-1.30	24
LU0073129206	UBS (Lux) Strategy	-21.83	16.73	-1.30	25
LU2001994727	UBAM Multif. Sel.	-27.18	17.33	-1.57	26

*The risk-free interest rate was assumed to be 0% when calculating the Sharpe ratio.

4.1.3 Rankings using Israelsen's Modified Sharpe Ratio

Israelsen (2005) added an exponent to the standard deviation (risk denominator), to improve the Sharpe ratio. When the period average excess return is positive, both formulas are algebraically identical — if applied once to the period mean. Why, then, do they still produce different numbers for the same fund over the same horizon?

The answer is that in practice the ratio is computed sub-period by sub-period (e.g., monthly), and the sub-period results are then averaged. This aggregation step exposes a fundamental asymmetry:

- Classical SR is linear: SR(mean) = mean(SR_t). Averaging monthly SRs gives the same result as computing SR from the period mean.
- Israelsen's ImSR is non-linear: the sign function $\varphi(r\_t) = r\_t/|r\_t|$ is evaluated each month. A month with a negative return switches the exponent to -1, flipping division into multiplication. ImSR(mean) ≠ mean(ImSR_t).

Table 3 reports the rankings based on Israelsen's Sharpe ratio (ImSR). As can be seen, the ImSR produces a more consistent ranking than the SR.

Table 3: Ranking if using the Israelsen's Sharpe ratio (lmSR)

ISIN	Equity Funds	Return %	Volatility %	ImSR	Ranking
AT0000A2HTY1	R&B Aktien Global Aktiv	-7.23	12.74	-92	1
AT0000722988	FWU TOP Trends	-6.79	13.64	-93	2
AT0000707682	Erste Best of World	-6.55	15.25	-100	3
AT0000754700	Hypo Dynamic Equity	-7.44	13.87	-103	4
AT0000A1VNX9	Faktorstrategie Akt. Glob.	-7.51	17.72	-133	5
IE00B537M394	GAM Star Comp. Glob.	-8.32	16.07	-134	6
AT0000746581	All Trends	-9.81	14.02	-138	7
AT0000A2B6F7	Hypo Vlbg. Weltportf.	-9.55	14.64	-140	8
AT0000703491	Piz Buin Global	-9.36	15.05	-141	9
DE0005314447	UniStrategie: Offensiv	-11.39	14.09	-161	10
DE0005326789	UniSelection: Global	-12.64	13.14	-166	11
AT0000708771	Apollo Dynamisch	-12.40	13.83	-171	12
AT0000801170	All World	-12.57	13.90	-175	13
AT0000811427	Equity s Best-Invest	-12.72	13.82	-176	14
AT0000A2GYP1	Nachhaltigkeitsstrat. Akt.	-12.55	14.98	-188	15
AT0000746524	FWU TOP spekulativ	-13.70	14.71	-202	16
LU0126855997	WWK Select Chance	-14.01	14.71	-206	17
AT0000784889	3 Banken Werte Growth	-12.52	16.76	-210	18
AT0000746516	FWU TOP offensiv	-14.18	15.10	-214	19
AT0000784830	3 Banken Aktienf.-Selek-	-17.13	13.87	-238	20
AT0000719893	Pro Invest Aktiv	-15.53	15.63	-243	21
AT0000731666	Portf. Next Generat. ZKB	-17.05	17.20	-293	22
AT0000817945	Aktienstrategie global	-19.63	15.10	-296	23
LU0073129206	UBS (Lux) Strategy Fund	-21.83	16.73	-365	24
LU2001994727	UBAM Multif. Sel. Trends	-27.81	17.33	-471	25
LU2123743341	BlackRock Mult.-Theme	-24.22	22.28	-540	26

4.1.4 Ranking based on the R-Sharpe ratio

The author's solution is to add a constant difference value (delta rpw; 1) to the numerator of the Sharpe ratio (excess return). The higher excess return divided by the standard deviation results in (1) positive numbers and (2) leaves the risk to return ratio compared to the SR unchanged. Using this modification results in an intuitive ranking.

Table 4: Ranking if using the R-Sharpe ratio (RSR)

ISIN	Equity Funds	Return %	Volatility %	RSR	Ranking
AT0000A2HTY1	R&B Aktien Global Aktiv	-7.23	12.74	1.65	1
AT0000722988	FWU TOP Trends	-6.79	13.64	1.57	2
AT0000754700	Hypo Dynamic Equity	-7.44	13.87	1.50	3
AT0000707682	Erste Best of World	-6.55	15.25	1.42	4
AT0000746581	All Trends	-9.81	14.02	1.31	5
AT0000A2B6F7	Hypo Vlbg. Weltportf.	-9.55	14.64	1.27	6
AT0000703491	Piz Buin Global	-9.36	15.05	1.25	7
IE00B537M394	GAM Star Comp. Glob.	-8.32	16.07	1.24	8
DE0005314447	UniStrategie: Offensiv	-11.39	14.09	1.19	9
DE0005326789	UniSelection: Global	-12.64	13.14	1.18	10
AT0000A1VNX9	Faktorstrategie Akt. Glob.	-7.51	17.72	1.17	11
AT0000708771	Apollo Dynamisch	-12.40	13.83	1.14	12
AT0000801170	All World	-12.57	13.90	1.12	13
AT0000811427	Equity s Best-Invest	-12.72	13.82	1.12	14
AT0000A2GYP1	Nachhaltigkeitsstrat.Akt.	-12.55	14.98	1.04	15
AT0000746524	FWU TOP spekulativ	-13.70	14.71	0.98	16
LU0126855997	WWK Select Chance	-14.01	14.71	0.96	17
AT0000784889	3 Banken Werte Growth	-12.52	16.76	0.93	18
AT0000746516	FWU TOP offensiv	-14.18	15.10	0.93	19
AT0000719893	Pro Invest Aktiv	-15.53	15.63	0.81	20
AT0000784830	3 Banken Aktienf.-Selek-	-17.13	13.87	0.80	21
AT0000731666	Portf. Next Generat. ZKB	-17.05	17.20	0.65	22
AT0000817945	Aktienstrategie global	-19.63	15.10	0.57	23
LU0073129206	UBS (Lux) Strategy Fund	-21.83	16.73	0.38	24
LU2123743341	BlackRock Mult.-Theme	-24.22	22.28	0.18	25
LU2001994727	UBAM Multif. Sel. Trends	-27.81	17.33	0.06	26

While Table 3 and Table 4 show a high degree of similarity in the rankings, there are nevertheless differences. The reason is that Israelsen's ratio depends on the distribution of returns within the period (not just on mean and standard deviation): two funds with identical mean ER and identical σ can receive different ImSR scores if they differ in how their monthly returns are arranged over time:

- A fund whose losses are concentrated in a few large negative months receives a more negative ImSR than a fund with the same total loss spread over many small negative months.
- This makes ImSR sensitive to the skewness and tail behaviour of the return distribution — a desirable property from a risk perspective, but one that does not arise in the classical SR.

Classical SR is fully determined by mean and standard deviation (the first two moments). Israelsen's ImSR implicitly incorporates higher-order information through the sign-conditional exponent.

Rating agencies, financial publications, and other practitioners that compile risk-adjusted performance rankings on the basis of the Sharpe ratio will find in the newly proposed R-Sharpe ratio an equivalent risk-adjusted measure capable of producing results that are directly comparable to those of the classical SR in periods of negative excess returns.

4.2 Results of downside performance measure

4.2.1 Modified (MVaR) Sharpe ratio

For calculating the MVaR, white noise error terms created from a GARCH-NIG model were used. Adjusting the excess return with the modified Value at Risk results in the performance measure MVaR Sharpe ratio (MSR). The analysis shows that the range of MSR results, from $-1.94\,\%$ to $-7.60\,\%$, was unexpectedly wide. The mean value was $-4.27\,\%$.

The biggest surprise was that a fund with volatility in the first quintile had with -7.22% the second worst result. Using the MSR, significant risk of loss that investors might sustain can be observed. Therefore, the variance result may be misleading because other funds may incur similarly large losses, which variance alone cannot detect.

4.2.2 Lower Partial Moments (LPM)

Prior to the LPM analysis, the fund returns were tested with the Jarque-Bera (JB) Test of their distributions. The JB test is a statistical test that uses skewness and kurtosis in the data to test whether there is a normal distribution and was proposed by Jarque and Bera (1980).

Normality is one of the assumptions for many statistical tests, like the t test or F test. A normal distribution has a skew of zero (i.e., it is perfectly symmetrical around the mean) and a kurtosis of three; kurtosis tells how much data is in the tails and gives an idea about how "peaked" the distribution is. It is not necessary to know the mean or the standard deviation for the data in order to run the test.

At a significance level of 5%, the hypothesis of normal distribution is rejected if $JB > 6$. This value was exceeded by 7 of 26 funds. It therefore makes sense to carry out a more sophisticated investigation with the Lower Partial Moments.

As Table 5 shows, there were major shifts in the ranking when considering the lower partial moments. The jump of one fund up 12 places (UniSelection Global) compared with Table 1 was the largest forward change. Its probability of losses lies at 52%. When losses occur, they are on average 0.374%. This fund benefited from its above-average right skewed distribution compared to the peer group.

Hence, use of the Lower partial moments becomes significant whenever the data does not conform to the assumptions underlying the Sharpe ratio (normality of returns).

Table 5: Peer group ranked by LPM of order 1

ISIN	Equity Funds	Return	Volatility %	LPM(0)	LPM(1)	LPM(2)	Ranking
AT0000A2HTY1	R&B Aktien Global	-7.23	12.74	50%	0.347	0.004	1
AT0000754700	Hypo Dynamic	-7.44	13.87	50%	0.367	0.005	2
DE0005326789	UniSelection: Global	-12.64	13.14	52%	0.374	0.004	3
AT0000722988	FWU TOP Trends	-6.79	13.64	52%	0.374	0.004	4
DE0005314447	UniStrategie: Offen-	-11.39	14.09	52%	0.378	0.005	5
AT0000801170	All World	-12.57	13.90	52%	0.379	0.005	6
AT0000708771	Apollo Dynamisch	-12.4	13.83	53%	0.388	0.005	7
AT0000746581	All Trends	-9.81	14.02	50%	0.391	0.005	8
AT0000707682	Erste Best of World	-6.55	15.25	52%	0.393	0.005	9
AT0000811427	Equity s Best-Invest	-12.72	13.82	56%	0.394	0.005	10
AT0000A2B6F7	Hypo Vlbg. Welt-	-9.55	14.64	52%	0.395	0.005	11
AT0000746524	FWU TOP spekula-	-13.7	14.71	52%	0.398	0.005	12
AT0000784830	3 Banken Aktienf.-	-17.13	13.87	56%	0.401	0.005	13
AT0000703491	Piz Buin Global	-9.36	15.05	53%	0.402	0.005	14
LU0126855997	WWK Select Chance	-14.01	14.71	51%	0.407	0.006	15
AT0000746516	FWU TOP offensiv	-14.18	15.10	51%	0.409	0.006	16
AT0000A2GYP1	Nachhaltigkeitsstrat.	-12.55	14.98	52%	0.420	0.006	17
IE00B537M394	GAM Star Comp.	-8.32	16.07	54%	0.423	0.006	18
AT0000817945	Aktienstrategie glo-	-19.63	15.10	50%	0.429	0.006	19
AT0000719893	Pro Invest Aktiv	-15.53	15.63	52%	0.431	0.006	20
AT0000784889	3 Banken Werte	-12.53	16.76	49%	0.456	0.007	21
AT0000731666	Portf. Next Generat.	-17.05	17.20	55%	0.458	0.007	22
AT0000A1VNX9	Faktorstrategie Akt.	-7.51	17.72	50%	0.469	0.007	23
LU0073129206	UBS (Lux) Strategy	-21.83	16.73	56%	0.475	0.007	24
LU2001994727	UBAM Multif. Seul.	-27.18	17.33	57%	0.506	0.007	25
LU2123743341	BlackRock Mult.-	-24.22	22.28	56%	0.603	0.012	26

4.2.3 Omega ratio

As presented in Keating and Shadwick (2002b) and Keating and Shadwick (2002a), for an asset whose return r has a cumulative probability distribution function F and ϑ is the target return threshold defining what is considered a gain versus a loss, the Omega ratio is defined as

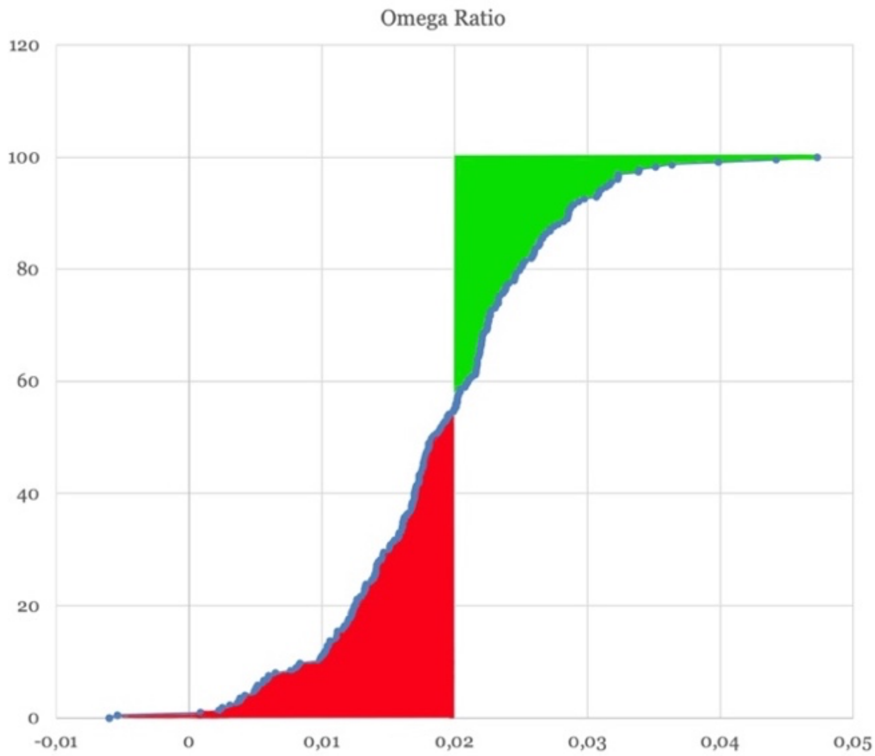
$$\Omega(\theta) = \frac{\int_{\theta}^{\infty} [1 - F(r)]\, dr}{\int_{-\infty}^{\theta} F\,(r)\, dr}$$

$$\mathrm{f}\,(r) = \frac{1}{\pi\sigma\left[1 + \left(\frac{r - \mu}{\sigma}\right)^{2}\right]}$$

$$\int_{\theta}^{A} [1 - F(r)]\, dr = \left[\frac{1}{2}\,\frac{r\, arctan(r)\; - \frac{ln\,(1 + r^{2})}{2}}{\pi}\right]_{\theta}^{A}$$

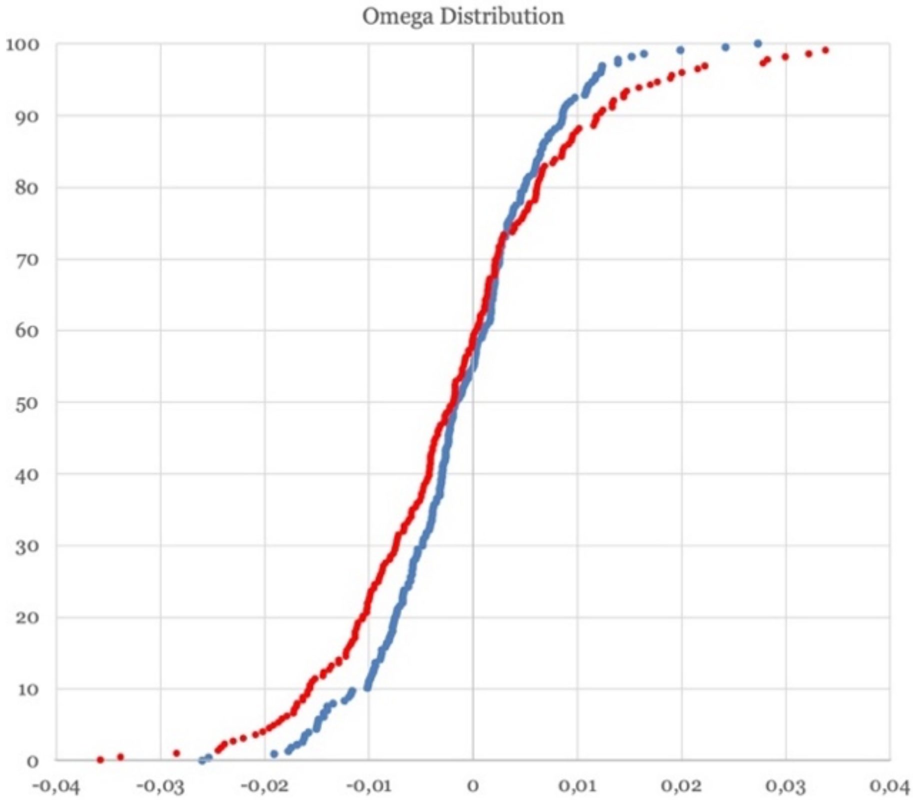
When ϑ is set to zero, the gain-loss ratio by Bernardo and Ledoit (2000) arises as a special case. The selling point of the omega ratio compared to the Sharpe ratio and other traditional risk ratios is that at first sight, it seems to depend on the entire return distribution through the cumulative probability distribution function F as well as not rely on any particular moments in terms of value and even existence, making it intellectually very attractive, as argued by Benhamou, Guez, and Paris (2019). Graphically, for a ϑ value of 2.00 percent and the cumulative distribution given by figure 1, the $\Omega(\vartheta)$ ratio is defined as the ratio of the red area over the green area.

Figure 1: Omega ratio ϑ = 2,00 Percent



The omega ratio can help to understand how well an investment strategy mitigates tail risk. Advisors will want to compare the omega ratios of a few funds to properly assess this. An example of this from the peer group: the fund with the lowest volatility (blue line) in Table 1 achieved a better loss limitation than the fund in position 26 (red line) over the one-year period under review. However, there is a cost to this: The price of protecting on the downside is to give up some of the returns on the upside. Figure 2 illustrates this trade-off.

Figure 2: Omega distribution of two funds



5. Conclusion

Although the Sharpe ratio is a popular performance measure, it has various limitations. When returns are negative or the return distributions deviate from normality, it may lead to unreasonable results. Negative Sharpe ratios arise when mutual funds underperform the risk-free rate. If investors pick the one with the highest Sharpe ratio, they are choosing the fund with the greatest volatility, rather than the least, since a negative number divided by a large number is greater than that negative number divided by a small number. Therefore, two performance measures that have refined the Sharpe ratio were used – Israelsen's Sharpe ratio (ImSR) and the R-Sharpe ratio (RSR) – to evaluate the performances of funds with negative returns. The empirical findings indicate that the ImSR and

RSR produce more consistent rankings than the Sharpe ratio. Due to its unchanged risk to return ratio compared to the SR, it can be concluded that the RSR produces – with the view of the Markowitz mean-variance model – correct risk-adjusted rankings if all or some returns of the peer group funds are negative. Rating agencies, financial magazines, and other practitioners constructing risk-adjusted performance rankings will find in the R-Sharpe ratio an equivalent measure. Three modern performance measures (MVaR Sharpe ratio, Lower Partial Moments and Omega ratio) were also examined to calculate downside risks. The results show that downside risks exist that would not be detected by the variance model.

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