The Power of Compound Interest: How Time, Reinvestment, and Day-Count Conventions Shape Terminal Wealth

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Abstract

This paper examines the economic mechanics of compound interest from the perspective of the averagely sophisticated retail investor. Starting from discrete compounding, it formally derives intra-year and continuous compounding via Euler's number e, systematises the day-count conventions used internationally (30/360, 30E/360, act/360, act/365, act/act), develops the complete formula-based arithmetic of the time value of money (future value, present value, interest rate, time, annuities, and perpetuities), and quantifies the effect of reinvestment on money market, bond, and equity investments using a unified set of hypothetical worked examples. In addition, it reviews the behavioural, psychological, and neurobiological foundations of the systematic underestimation of exponential growth (exponential growth bias, present bias), the savings-plan mathematics of the time factor, and the empirical evidence on market timing.

The results highlight the dominant role of the investment horizon and of reinvestment. For an equity investment of 100,000 euro with a total return of 7 percent per annum, an accumulating investor reaches terminal wealth of 386,968 euro after 20 years, whereas an investor who consumes the annual distribution of 2.5 percent ends up with only 319,600 euro including all amounts withdrawn — a shortfall of 21.1 percent. For a ten-year coupon bond, up to one quarter of total interest income is attributable to interest on interest, depending on the reinvestment rate. Under uncertainty, volatility dampens capital growth according to $g \approx \mu - \sigma^2/2$ (volatility drag), while the wealth multiplier $(1+g)^T$ grows convexly in the horizon. Behavioural economics explains why the effect nevertheless remains unexploited: households systematically underestimate exponential growth, and moving from full bias to accurate perception is associated with 55 to 90 percent higher accumulated assets (Levy and Tasoff, 2016). The savings-plan mathematics

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shows that the horizon dominates the interest rate as a lever — the elasticities of terminal wealth grow linearly in T — and the timing evidence shows that merely missing the ten best trading days costs roughly half of terminal wealth (Estrada, 2008), while market timing only pays beyond a forecasting accuracy of about 74 percent (Sharpe, 1975).

The findings imply that time in the market, consistent reinvestment, cost control, and knowledge of day-count conventions are the essential levers by which retail investors can harness the power of compound interest. The paper is intended as a didactic synthesis of the literature, not as investment advice.

Keywords: compound interest · accumulation · cost averaging · day-count conventions · volatility drag · investment horizon · exponential growth bias · market timing.

JEL codes: D14, D91, E43, G11, G41, G51, G53.

Non-Technical Summary

A capital investment generates income — interest, coupons, or dividends. If this income is not withdrawn but reinvested, it will itself generate income in the future. This interest on interest is the compound interest effect. It appears inconspicuous at first, yet unfolds enormous power over long periods: constant percentage growth turns into exponential growth of wealth.[2]

The core question of this paper is how large this effect actually is across the common asset classes and what it depends on. The answer, in plain terms: it depends primarily on three quantities — the level of the return, the length of the investment period, and the consistency with which income is reinvested. Of these three, time is the strongest lever, because wealth grows not evenly but at an accelerating pace with every additional year.

Methodologically, the paper presents the computational variants of compounding — from annual through monthly to so-called continuous compounding, which rests on Euler's number e — and explains the internationally differing conventions for counting interest days (such as 30/360 or act/365), which lead to different interest amounts depending on country and market segment. All statements are anchored in fully worked, hypothetical examples.

The central example compares two investors over 20 years: both invest 100,000 euro in the same equities with a total return of 7 percent per annum. Investor A reinvests all distributions (he accumulates), investor B has 2.5 percent paid out annually and consumes these amounts. In the end, investor A owns roughly 387,000 euro; investor B, including all amounts paid out, reaches only about 320,000 euro. The difference of a good 67,000 euro, or 21 percent, is solely the compound interest effect of the reinvested distributions.

Three further findings round out the picture. First, people systematically underestimate exponential growth because human thinking follows linear patterns — compound interest therefore seems disappointingly inconspicuous at first and only surprises in later years; this knowledge supports perseverance. Second, time beats the interest rate: an early start with small amounts outperforms a late start with large amounts — in the example, a saver reaches 96 percent of the terminal wealth with one third of the contributions, merely because she started ten years earlier. Third, time in the market matters more than the right moment: missing only the ten strongest trading days over decades costs roughly half of terminal wealth.

The practical conclusion: start early, reinvest income, keep costs low, and endure fluctuations. Observing these four rules makes time — the strongest determinant of long-term wealth accumulation — effective.

[2] All numerical examples in this paper are hypothetical, computed before taxes and costs, and do not constitute investment, legal, or tax advice.

The arithmetic for this is supplied by the time value of money: with a handful of formulas one can compute what today's money will become (future value), what future money is worth today (present value), what average return lies between two wealth levels, and how long a doubling takes. This paper assembles all of these formulas with short explanations; once worked through, they permit independent verification of any savings product, any loan, and any quoted return.

Finally, the order of the levers matters: the early start achieves the most, followed by the size of the regular instalment and consistent reinvestment, then the costs of the product — and only last the hunt for the highest return. This ranking is reassuring, because the most important levers lie entirely in one's own hands.

Contents

1 Introduction

Few mechanisms in finance are at once as simple and as consequential as compound interest. If the income from an investment is not withdrawn but added to the capital, wealth grows not linearly but exponentially: tomorrow's return is earned on today's return as well. Over short periods, the difference between simple interest and compound interest is small; over decades, it determines a multiple of terminal wealth. Nevertheless, household finance research regularly documents that precisely this exponential growth is systematically underestimated — a phenomenon known as exponential growth bias.[3]

Historically, interest is as old as writing itself: interest-bearing loans are documented for Mesopotamia from the third millennium BC onwards, and the Code of Hammurabi already regulated interest rate ceilings (Homer and Sylla, 2005). Compound interest was recognised early as a distinct — and ambivalent — force: Babylonian clay tablets contain exercises on the doubling time of capital at interest, medieval canonists wrestled with the usury prohibition, and the commercial practice of the Italian city-states made interest arithmetic the foundation of modern credit and capital markets. The present paper thus stands in a long tradition of measuring exponential capital growth — with the difference that it addresses today's retail investor.

The formal foundations of compound interest reach far back: Pacioli (1494) documents, with the rule of 72, an early approximation for doubling times; Jacob Bernoulli, analysing continuous compounding, encountered the limit of $(1 + 1/n)^n$; and Euler (1748) gave the resulting constant e its name. Böhm-Bawerk (1889) grounded interest in time preference, Fisher (1930) anchored it as the price of time in economic theory, and Williams (1938) made the present value calculus the foundation of security valuation. While modern portfolio theory since Markowitz (1952) considers return and risk in a single-period setting, Kelly (1956), Latané (1959), Samuelson (1969), and Merton (1969) placed the growth of capital over many periods at the centre of the analysis. Messmore (1995) coined the term variance drain for the difference between the arithmetic and the geometric mean; Constantinides (1979), Statman (1995), and Dichev (2007) examined the properties of instalment investing. Stango and Zinman (2009), finally, show that households systematically underestimate exponential growth and therefore save too little.

The contribution of this paper lies in its didactic synthesis: it combines the formal presentation of compounding variants and day-count conventions with fully worked, unified examples for money market, bond, and equity investments, and quantifies the power of compound interest where retail investors actually encounter it — in the interest credit on a call money account, the reinvestment of coupons, the choice between

[3] The dictum frequently attributed to Albert Einstein — that compound interest is the eighth wonder of the world — is unverified and considered apocryphal; the effect it describes is nonetheless real.

accumulating and distributing funds, and instalment saving. All examples are hypothetical, computed before taxes and costs, and serve illustrative purposes only.

Methodologically, the paper is a normative-didactic synthesis: it derives rules of action from formal theory and replicated evidence without presenting econometric estimates of its own; where empirical magnitudes are cited, they stem from the referenced primary studies. This design follows the tradition of the household finance literature, which derives normative benchmarks from theory and measures observed investor behaviour against them — the gap between the two is the proper subject of Sections 8 to 10.

The paper is structured as follows. Section 1 (the present section) serves as the introduction, frames the research question, and surveys the prior literature. Section 2 develops the computational variants of compound interest, from discrete through intra-year to continuous compounding based on Euler's number. Section 3 presents the complete arithmetic of the time value of money — future and present value, interest rate, time, annuities, and perpetuities, with all formulas. Section 4 systematises the day-count conventions in international use. Section 5 quantifies the compound interest effect for money market and bond products, Section 6 for equities and equity funds, including accumulation and the cost-average effect. Section 7 addresses capital growth under uncertainty through volatility drag and horizon effects. Section 8 turns to the psychology and biology of compound interest — exponential growth bias and present bias. Section 9 uses savings-plan mathematics to show that the time factor dominates the interest rate as a lever, and Section 10 confronts market timing with the empirical evidence. Section 11 summarises the results and derives practical recommendations.

2 Computational Variants: From Simple Interest to Euler's Number

2.1 Simple Interest and Discrete Compounding

Under simple interest, interest is always paid on the initial capital K0 only; wealth grows linearly according to $K_T = K0 \cdot (1 + i \cdot T)$. Under compound interest, the interest is added to the capital and earns interest itself in the following period; wealth grows geometrically according to $K_T = K0 \cdot (1 + i)^T$. The difference is small at first and then grows at an accelerating pace: at an interest rate of 4 percent, simple interest yields a total return of 40.00 percent after ten years, compound interest 48.02 percent — the excess of a good 8 percentage points is exclusively interest on interest.[4]

The geometric growth equation implies the doubling time $T^* = \ln 2 / \ln(1 + i)$. At 6 percent, capital doubles every 11.9 years; at 2 percent, only every 35.0 years. The rule of

[4] The rule of 72 — doubling time ≈ 72 divided by the interest rate in percent — already appears in Pacioli (1494); it approximates the exact solution $T^* = \ln 2 / \ln(1 + i)$.

72, widely used as a heuristic ($T^* \approx 72$ / interest rate in percent), delivers remarkably accurate approximations of 12.0 and 36.0 years, respectively.

The central property of compound interest is its convexity in time: each additional period raises terminal wealth by a larger absolute amount than the preceding one. It is precisely this property that investors underestimate (Stango and Zinman, 2009), and it explains why starting early is more effective than raising the savings rate later.

The computational rules themselves are older than their theoretical penetration: the interest tables of the early modern period — in whose tradition Pacioli (1494) stands — allowed merchants to read off terminal values without repeated multiplication (Homer and Sylla, 2005). The sign symmetry of the mechanism is also noteworthy: at negative interest rates, as the euro area experienced between 2014 and 2022, the same exponent reduces wealth. Compound interest has no preference for signs — it amplifies whatever it is given: income, costs, or losses.

The same symmetry holds on the liability side of the household: revolving consumer credit and credit card balances compound against the borrower by identical mechanics, frequently with monthly capitalisation and double-digit effective rates. An unpaid balance doubles in roughly four years at an 18 percent effective rate — the rule of 72 works as reliably on debt as on assets. The first application of compound arithmetic in the household context is therefore not the investment but the deleveraging decision: hardly any safe investment earns the return that repaying expensive consumer credit yields risk-free.

2.2 Intra-Year Compounding and the Effective Annual Rate

If interest is credited m times per year, the growth equation reads $K_T = K0 \cdot (1 + i/m)^{m \cdot T}$. The nominal annual rate i must then be distinguished from the effective annual rate $i_{eff} = (1 + i/m)^m - 1$, which incorporates intra-year compounding and is the relevant measure for comparing offers with different crediting frequencies. Table 1 shows the effect for an investment of 10,000 euro at a nominal rate of 4 percent over ten years.

Table 1: Terminal value of 10,000 euro at a 4 percent nominal rate over ten years by compounding frequency

Compounding	Terminal value (€)	Effective annual rate
annual ($m = 1$)	14,802.44	4.0000%
semi-annual ($m = 2$)	14,859.47	4.0400%
quarterly ($m = 4$)	14,888.64	4.0604%
monthly ($m = 12$)	14,908.33	4.0742%
daily ($m = 365$)	14,917.92	4.0808%
continuous ($m \to \infty$)	14,918.25	4.0811%

Source: own calculations; hypothetical example before taxes and costs.

Table 1 illustrates two points. First, every increase in crediting frequency raises terminal wealth — intra-year compounding is real. Second, the marginal benefit of frequency diminishes rapidly: after ten years, only 115.81 euro, or 0.78 percent of the terminal value, separates annual from continuous compounding. For terminal wealth, return and investment horizon are orders of magnitude more important than crediting frequency; nevertheless, checking the effective annual rate belongs to the basic knowledge of the informed investor.

The practical importance of the effective rate is also anchored in regulation: the european Consumer Credit Directive obliges providers to state the annual percentage rate of charge, and the US Truth in Savings Act prescribes disclosure of the annual percentage yield. The legislator thereby codifies precisely the lesson of the relation $i_{eff} = (1 + i/m)^m - 1$: without a uniform effective rate disclosure, offers with different crediting frequencies and conventions would be systematically incomparable for consumers.

The frequency effect scales with the level of interest rates: the difference between nominal and effective annual rates grows approximately quadratically in the rate — at a 1 percent nominal rate, less than half a basis point separates annual from monthly crediting, while at 12 percent (the range of much consumer credit) the gap is already about 68 basis points. Precisely where households most often encounter nominal quotes, the difference is largest — a further argument for consistently working with effective annual rates.

2.3 Continuous Compounding and Euler's Number

Letting the number of interest credits grow without bound ($m \to \infty$), $(1 + i/m)^m$ converges to e^i, where $e = \lim (1 + 1/n)^n = 2.71828...$ denotes Euler's number. Continuous compounding thus leads to the elegant growth equation $K_T = K0 \cdot e^{i \cdot T}$. Historically, Euler's number arose directly from the question of compound interest — it is, quite literally, the number of compound interest.[5]

[5] The limit $\lim (1 + 1/n)^n$ was found by Jacob Bernoulli in 1683 while studying continuous compounding; the notation e is due to Euler (1748).

For financial mathematics, the continuous view is more than a curiosity: the continuously compounded return $r = \ln(1 + i)$ is additive over time, so that multi-period returns can be written as simple sums. This property underlies the logarithmic returns of empirical capital market research as well as the continuous-time models of option pricing and portfolio theory (Merton, 1969).

For retail investors, it suffices to recognise that continuous compounding marks the upper bound of the frequency effect and at the same time provides the natural arithmetic for long-term growth: growth rates add up in the logarithmic world, which is precisely why returns, costs, and inflation are compared most cleanly in continuous terms over long horizons.

Beyond investment arithmetic, continuous compounding is the foundation of modern valuation theory: the option pricing model of Black and Scholes (1973) discounts with the factor e^{-rT}, the term structure models of fixed income analytics work throughout with continuously compounded spot rates, and logarithmic returns are the standard of empirical capital market research because they are additive over time and approximately normally distributed. Euler's number thus connects the passbook arithmetic of the retail investor with the valuation technology of the investment bank — it is literally the same mathematics.

3 The Time Value of Money: The Arithmetic of Compound Interest

3.1 The Four Fundamental Quantities: Future Value, Present Value, Interest Rate, and Time

The time value of money is the central idea of finance: a euro today is worth more than a euro tomorrow, because it can be invested at interest until tomorrow — and compound interest is the mechanism that builds up this difference in value over time. Böhm-Bawerk (1889) grounded interest in the systematic preference for present over future goods; Fisher (1930) formalised it as the price of time. The entire arithmetic of the time value of money consists of four quantities — present value PV, future value FV, interest rate i, and time n — linked by a single equation; the following formulas are merely solutions of that equation for one unknown at a time. All four forms are equivalent: they describe the same compounding mechanism, read once forwards and once backwards through time.[6]

$$FV = PV \cdot (1 + i)^n \tag{1}$$

[6] As early as 1202, Leonardo of Pisa (Fibonacci) solved present value problems of commercial trade in the Liber Abaci; Goetzmann (2004) sees in this the beginning of modern financial mathematics.

Equation (1) is the future value formula — the fundamental equation of compound interest: an amount PV invested today grows at rate i over n periods to the future value FV. It answers the question of what today's money will become.

$$PV = FV \,/\, (1 + i)^n \tag{2}$$

Equation (2) is the present value formula (discounting) — the same equation solved for PV: a future payment FV is worth only PV today. Here the power of compound interest appears in reverse, for distant payments lose considerably in present value through discounting: 100,000 euro in 30 years is worth only 23,138 euro today at 5 percent.

$$i = (FV \,/\, PV)^{1/n} - 1 \tag{3}$$

Equation (3) solves for the interest rate and yields the average geometric annual return (CAGR, compound annual growth rate) between two wealth levels — the correct measure for comparing investments over periods of different length.

$$n = ln(FV \,/\, PV) \,/\, ln(1 + i) \tag{4}$$

Equation (4) solves for the time: how long does it take for PV to become the target amount FV? For FV/PV = 2 it yields the doubling time of Section 2, whose approximation is the rule of 72.

The four fundamental quantities form a closed system: every investment decision — from the call money account through the bond to the equity — is implicitly an application of equations (1) to (4), and every later formula in this paper, from the annuity formula of the savings plan to the growth rate $g \approx \mu - \sigma^2/2$, is derived from them. Compound interest is the engine; the time value of money is its arithmetic.

The present value logic reaches beyond financial assets: a household's human capital — the present value of expected future labour income — is an application of equation (2) and, for young investors, regularly the largest asset. From this perspective, the common recommendation to hold higher equity shares at younger ages also becomes intelligible: where the bulk of total wealth is held as bond-like human capital, riskier financial assets merely diversify the overall portfolio (Merton, 1969; Samuelson, 1969).

In market practice, a whole term structure takes the place of the single rate i: for every maturity t there is a separate spot rate, and the discount factor $1/(1 + i_t)^t$ of a payment is the price of a zero-coupon bond of that maturity. The equations of this section apply unchanged — each payment is simply valued with the factor of its own maturity. For the retail investor it suffices to recognise that the market quotes a separate price of time for every horizon, which is why the valuation of long-dated payments is particularly sensitive to interest rates.

3.2 Annuities, Perpetuities, and the Present Value of Cash Flow Streams

Real cash flow streams rarely consist of a single payment: savings plans, loan instalments, coupons, and dividends are annuities — sequences of equal periodic payments R. Their future and present values arise from summing the geometric series of the individual payments and constitute the arithmetic behind almost every financial product:[7]

$$FV = R \cdot [(1 + i)^n - 1] / i \tag{5}$$

Equation (5) is the future value of an ordinary annuity (payment at period end): the terminal value of a savings plan with constant instalment R — the formula behind Table 8 and the two-saver experiment in Section 9.

$$FV = R \cdot [(1 + i)^n - 1] / i \cdot (1 + i) \tag{6}$$

Equation (6) is the future value of an annuity due (payment at period start): each instalment compounds one period longer — the additional factor (1 + i) is the compound interest of precisely that one additional year.

$$PV = R \cdot [1 - (1 + i)^{-n}] / i \tag{7}$$

Equation (7) is the present value of an annuity: today's value of n future instalments. It is the foundation of loan and annuity arithmetic — solved for R it yields the annuity of a loan — and of the valuation of coupon bonds.

$$PV = R / i \tag{8}$$

Equation (8) is the perpetuity: the present value of an infinite, constant stream of payments. That infinitely many payments have a finite value today is perhaps the most impressive demonstration of the discounting power of compound interest.

$$PV = R / (i - g), \quad i > g \tag{9}$$

Equation (9) is the growing perpetuity (Gordon growth model): the present value of a payment stream growing at rate g, such as a dividend. It condenses fundamental equity valuation in the tradition of Williams (1938) and Gordon and Shapiro (1956) into a single line.

$$1 + r = (1 + i) / (1 + \pi) \tag{10}$$

[7] Williams (1938) founded fundamental equity valuation on the present value calculus; the growth model in equation (9) is known, after Gordon and Shapiro (1956), as the Gordon growth model.

Equation (10) is the Fisher equation (Fisher, 1930): it decomposes the nominal rate i into the real rate r and the inflation rate π. For long-term wealth accumulation, only real compound interest counts — inflation is compound interest with the opposite sign.

This closes the circle to the title of this paper: equations (1) to (10) are ten readings of the same force. The future value shows the power of compound interest as the ally of the patient saver; the present value shows the same power as an incorruptible yardstick that reduces future promises to their present core; and the Fisher equation is a reminder that this power must be earned in real terms, after inflation. The arithmetic of the time value of money permits converting every investment, every loan, and every distribution into the same reference unit: today's euros.

The range of application of these formulas is considerable: equation (7) prices every instalment loan and every mortgage, equation (8) the claim to a lifelong payout as well as the rental income of a property, and equation (9) underpins the spending policies of foundations and university endowments that wish to finance a permanently constant real withdrawal from a growing capital stock. In each of these cases the same quantity decides between sustainability and depletion: the ratio of the withdrawal rate to the growth rate of capital — compound interest as a budget constraint.

3.3 Net Present Value and Internal Rate of Return: Decision Rules of the Time Value

For cash flow streams with unequal payments CF_t, the net present value generalises the present value calculus into the central decision rule of investment theory:[8]

$$NPV = \Sigma\, CF_t \,/\, (1 + i)^t, \quad t = 0, \ldots, n \tag{11}$$

Equation (11) is the net present value: the sum of all inflows and outflows of a project or security, discounted at the hurdle rate i. The decision rule reads: investments with positive NPV increase wealth, those with negative NPV diminish it — the compounding of the hurdle rate is the yardstick against which every alternative must be measured.

$$0 = \Sigma\, CF_t \,/\, (1 + IRR)^t \tag{12}$$

Equation (12) defines the internal rate of return as the rate at which the net present value equals zero — the return implicit in the cash flow stream. The yield to maturity of a bond (Section 5) and the dollar-weighted investor return in Dichev (2007) are applications of this concept.

[8] By the separation theorem of Fisher (1930) and Hirshleifer (1958), investment decision and consumption preference are separable: in a perfect capital market, maximising net present value is optimal for every investor — irrespective of time preference.

The internal rate of return is also a call for care: for cash flow streams with multiple sign changes, equation (12) can have several solutions, and the IRR rule implicitly assumes reinvestment of all inflows at the internal rate itself — an assumption that Section 5.2 makes concrete as reinvestment risk. For retail investors, a simple check follows: return figures of products with complex payment profiles should always be examined for the reinvestment rate they conceal. Where the NPV and IRR rules conflict, precedence belongs to the net present value (Hirshleifer, 1958).

Taken together, equations (1) to (12) also form the scaffolding of fundamental security analysis: the discounted cash flow model of equity valuation is nothing but equation (11) applied to a company's expected free cash flows, and the Gordon growth model of equation (9) is its closed-form special case. The mechanism of compound interest also explains why small changes in the discount rate trigger large changes in valuations — the convexity of the present value function is the flip side of the convexity of the terminal value.

4 Day-Count Conventions in International Comparison

4.1 A Taxonomy of Day Counting

Interest for fractions of a year is computed using the accrual factor days/basis. How days and basis are to be counted is governed by market conventions that have evolved differently across countries and market segments: the 30/360 method (German commercial interest calculation, US bond basis) assumes twelve months of 30 days; the eurobond variant 30E/360 treats month-end dates differently; act/360 counts actual days over a 360-day basis; act/365 over a 365-day basis (occasionally encountered as a 30/365 hybrid); and act/act uses actual days in both numerator and denominator.[9]

The mapping to market segments follows firm conventions: German federal bonds have accrued on act/act (ICMA) since 1999, US corporate bonds predominantly on 30/360, the euro money market — including the reference rate €STR — on act/360, and the sterling money market on act/365. A comparison of bonds or money market products across borders therefore requires correcting for the convention; otherwise unlike quantities are compared.

Business-day conventions come on top of day counting: if a payment date falls on a weekend or holiday, rules such as Following or Modified Following shift the payment to the next — or, at month-end, the preceding — business day, which changes the interest period and hence the amount once more. Leap years, too, are treated differently: act/act under ISDA (2006) divides the days falling into a leap year by 366, whereas the ICMA variant relates the actual days to the respective coupon period — in detail, both methods can deliver different factors for the same period.

[9] The authoritative definitions of the conventions are found in the 2006 ISDA Definitions and in Rule 251 of the International Capital Market Association (ICMA).

For practice, a simple verification routine follows: the governing convention is stated in a security's terms and conditions or term sheet and in the price schedules of deposit products; together with coupon, maturity, and payment frequency, it is one of the four pieces of information without which no interest amount can be recomputed. Verification of accrued interest, term deposit statements, or fund income therefore always starts with determining the day count; all further steps then follow mechanically from the formulas of Section 3.

4.2 The Economic Impact of Conventions

Table 2 shows the economic impact of day counting for a semi-annual period from 31 January to 31 July 2026 (181 actual days) on a notional of 1,000,000 euro with a coupon of 5 percent.

Table 2: Accrued interest for a semi-annual period by day-count convention (notional EUR 1,000,000 euro, coupon 5 percent, 31 January to 31 July 2026)

Convention	Days	Accrual factor	Interest (€)
30/360	180	0.500000	25,000.00
30E/360	180	0.500000	25,000.00
act/360	181	0.502778	25,138.89
act/365	181	0.495890	24,794.52
act/act (ISDA)	181	0.495890	24,794.52

Source: own calculations; hypothetical example before taxes and costs.

The most and the least favourable counting methods are 344.37 euro apart, or roughly 1.4 percent of the interest amount — purely by convention, with coupon and period unchanged. Institutionally, this is immediately material at large notionals; retail investors encounter the effect in accrued interest, term deposits, and money market funds, and wherever nominal rates quoted on different conventions are compared. Because compound interest builds on credited interest, convention differences propagate multiplicatively over long horizons — the correct basis of comparison is always the effective annual rate on a uniform convention.

A related source of error are quotation conventions: money market paper such as US Treasury bills is quoted on a discount basis whose rate lies systematically below the true investment return, and rates on an act/360 basis appear optically lower than equivalent rates on act/365. A comparison of yields across market segments therefore requires harmonising the quotation basis alongside the day count — otherwise the power of compound interest is sought in the wrong place: in apparent yield differences that are pure artefacts of convention.

Historically, the 360-day bases are explained by computational practice: a year of twelve months of 30 days permitted clean divisions in pre-machine bookkeeping and ultimately traces back to the Babylonian sexagesimal system (Homer and Sylla, 2005).

That these conventions have survived the electronification of markets is a lesson in institutional path dependence — and the reason why every international yield comparison must begin with the inconspicuous question of how the days are counted.

5 Compound Interest in Money Market and Bond Products

5.1 Money Market: The Frequency of Interest Crediting

In call money accounts, term deposits, and money market funds, compound interest appears in its purest form, because income is credited regularly and automatically earns further interest. The crediting frequency is decisive: a call money account with a 3 percent nominal rate and monthly crediting achieves an effective annual rate of 3.04 percent; with annual crediting it remains at 3.00 percent. Money market funds accumulate the daily accruing income in the unit price and thereby realise intra-year compounding in full.[10]

Over short periods the effect is modest, but over years it becomes visible — and it also operates when rolling over term deposits: reinvesting interest and principal in full at maturity earns future interest on the interest received as well; rolling over only the principal and leaving the interest in the current account interrupts the compounding chain at its most effective point, the beginning.

The sign regime also shapes the effect: in the euro area's negative rate phase from 2014 to 2022 the mechanism reversed — custody fees acted as negative compound interest, and money market investments lost value in nominal as well as real terms. For the money market, what matters economically is therefore less the level of the nominal rate than the real, inflation-adjusted effective rate of equation (10); over long periods this has historically often been close to zero or negative for money market investments (Dimson, Marsh, and Staunton, 2002).

5.2 Bonds: Coupon Reinvestment and Interest on Interest

With coupon bonds, compound interest does not arise automatically: the coupons are paid out to the investor and must be actively reinvested. The terminal value of the coupons follows the future value formula of an annuity, $\mathrm{FV} = \mathrm{C} \cdot ((1 + \mathrm{r})^{\mathrm{T}} - 1)/\mathrm{r}$, where C denotes the coupon and r the reinvestment rate. Table 3 decomposes the terminal coupon value of a ten-year bond (notional 100,000 euro, coupon 4 percent) into the sum of the coupons and the interest on interest.[11]

[10] The euro reference rate €STR is quoted — like the euro money market as a whole — on an act/360 basis.
[11] The yield to maturity implicitly assumes that all coupons are reinvested at the internal rate of return until maturity; if the reinvestment rate falls, the quoted yield is missed (reinvestment risk).

Table 3: Terminal value of the coupons of a ten-year bond (notional 100,000 euro, coupon 4 percent) as a function of the reinvestment rate

Reinvestment rate	Terminal coupon value (€)	of which interest on interest (€)	Total wealth (€)
0%	40,000.00	0.00	140,000.00
2%	43,798.88	3,798.88	143,798.88
4%	48,024.43	8,024.43	148,024.43
6%	52,723.18	12,723.18	152,723.18

Source: own calculations; hypothetical example before taxes and costs.

At a reinvestment rate of 4 percent, 8,024 euro or 16.7 percent of the terminal coupon value stems from interest on interest; at 6 percent it is already 24.1 percent — nearly a quarter of total interest income. At the same time, the table exposes reinvestment risk: if reinvestment is omitted (r = 0), up to 12,723 euro is forgone. Zero-coupon bonds and accumulating bond and money market funds solve this problem constructively by shifting reinvestment into the product — economically, the zero-coupon bond is nothing but securitised compound interest at the rate fixed at purchase.

The magnitude of the interest-on-interest share grows dramatically with maturity and rate level: in their classic analysis, Homer and Leibowitz (1972) show that for long-dated bonds at higher rate levels interest on interest can account for more than half of the total return of a bond held to maturity — a finding that elevates reinvestment risk from a side issue to the central return component of the bond portfolio. Over long horizons, the seemingly conservative coupon bond is in truth a bet on future reinvestment rates.

5.3 Duration and Immunisation: Price Risk and Reinvestment Risk in Balance

Coupon reinvestment and the price risk of a bond are two sides of the same interest rate sensitivity, and their point of balance can be determined exactly. The duration introduced by Macaulay (1938) is the present-value-weighted average time to a bond's payments:[12]

$$D = [\Sigma\, t \cdot CF_t \,/\, (1 + i)^t] \,/\, P \tag{13}$$

Equation (13) is the Macaulay duration: the present-value-weighted average of the payment dates of a bond with price P. For the ten-year 4 percent bond of Table 3 it equals 8.44 years — well below the remaining maturity, because the coupons return capital early.

$$\Delta P \,/\, P \approx -\, D \,/\, (1 + i) \cdot \Delta i \tag{14}$$

Equation (14) translates duration into price sensitivity (modified duration D/(1+i)): if the rate rises by one percentage point, the bond price falls by approximately D/(1+i) percent — around 8.1 percent in the example.

[12] The duration concept goes back to Macaulay (1938); the immunisation property was formalised by Fisher and Weil (1971).

$$\Delta P / P \approx - D / (1 + i) \cdot \Delta i + \tfrac{1}{2} \cdot C \cdot (\Delta i)^2 \quad (15)$$

Equation (15) augments the linear approximation by the convexity C, the second derivative of the price-yield function: because the present value is convex in the rate, prices fall less when rates rise and rise more when rates fall than duration alone indicates. Convexity is thus — like compound interest itself — a consequence of the exponential discount factor.

The practical point is immunisation: rising rates lower the price but simultaneously raise the reinvestment income of the coupons — at an investment horizon equal to the duration, the two effects approximately offset (Fisher and Weil, 1971). Aligning the investment horizon with the duration of the bond portfolio offsets the compounding of reinvestment against price risk: terminal wealth becomes insensitive to rate changes to first order. Duration arithmetic thus provides the bond-specific answer to the guiding question of this paper — how the power of compound interest can be harvested in a plannable way.

In practice, retail investors implement the immunisation idea with simple means: a bond ladder spreads maturities evenly across the years, so that capital is continually reinvested at the prevailing rate and the average duration remains approximately constant; fixed-maturity bond funds bundle the same mechanism into a single product. Both approaches convert interest rate risk into a plannable reinvestment path — compound interest turns from a disturbance into a design parameter.

6 Compound Interest in Equities and Equity Funds

6.1 Accumulation versus Distribution: A 20-Year Comparison

In equities and equity funds, the reinvestment of dividends takes over the role of the interest credit. The following example isolates its contribution: two investors each invest 100,000 euro in the same equity portfolio with a total return of 7 percent per annum, composed of 4.5 percent price appreciation and 2.5 percent distribution. Investor A reinvests all distributions immediately (accumulation); investor B has the distribution paid out annually and consumes it. Table 4 juxtaposes the wealth positions after 20 years.[13]

[13] In Germany and Austria, withholding and capital gains taxes on distributions (in Germany also the advance lump-sum tax on accumulating funds) reduce the effect; the advantage of early reinvestment remains in substance.

Table 4: Accumulation versus distribution over 20 years (initial capital 100,000 euro, total return 7 percent p.a., of which 2.5 percent distribution)

Position	Investor A (accumulating)	Investor B (distributing)
Portfolio value after 20 years	386,968.45	241,171.40
Distributions consumed (nominal)	—	78,428.56
Total wealth including withdrawals	386,968.45	319,599.96
Shortfall versus accumulation	—	67,368.49 (21.1%)

Source: own calculations; hypothetical example before taxes and costs.

Although investor B receives all distributions and loses nothing, his total wealth falls short of the accumulator's by 67,368 euro or 21.1 percent. The difference is pure compound interest: every dividend consumed forgoes all the income it would still have earned until the horizon — a distribution of 2,500 euro withdrawn in the first year would have grown to 9,041 euro by year 20. In fund investing, accumulating share classes replicate this mechanism automatically; with distributing classes, the investor must organise the reinvestment himself and bears transaction costs and behavioural risks in doing so.

The long-run evidence underlines the weight of the distribution component: over periods of several decades, a substantial part of the real total return of equity markets stems from reinvested dividends rather than price appreciation (Siegel, 2002; Dimson, Marsh, and Staunton, 2002). Dividend reinvestment programmes and accumulating index funds institutionalise this insight by entrusting reinvestment to automatism rather than discipline.

The tax deferral of accumulation can be quantified: if the annual return is taxed continuously at rate s, capital grows at $(1 + \mathrm{i}\cdot(1 - \mathrm{s}))^{\mathrm{T}}$; if only the terminal value is taxed, $(1 + \mathrm{i})^{\mathrm{T}} \cdot (1 - \mathrm{s}) + \mathrm{s}$ remains. At a 7 percent return, 30 years, and a tax rate of 26.375 percent, deferred taxation delivers 5.87 euros per euro invested, continuous taxation only 4.52 euros — an advantage of roughly 30 percent purely from the compounding of the deferred tax. Real tax regimes lie between the two poles; the direction of the effect is unaffected.

Why investors nevertheless cherish distributions is explained by behavioural economics through mental accounting: dividends are booked as spendable income, capital gains as untouchable principal — a separation that is economically unfounded but shapes real consumption and reinvestment decisions (Shefrin and Statman, 1984). For long-term wealth accumulation this preference is expensive: it systematises precisely the withdrawal whose opportunity costs Table 4 quantifies.

6.2 Instalment Saving and the Cost-Average Effect

When saving through an investment plan, compound interest meets a second mechanism, the cost-average effect: investing a fixed amount regularly acquires more units at low prices and fewer at high prices. The average purchase price then equals the harmonic mean of the prices and is therefore mathematically always below their arithmetic mean. Table 5 illustrates this for a plan of 300 euro per month over six months with fluctuating prices.[14]

Table 5: Cost-average effect for an investment plan of 300 euro per month over six months

Month	Price (€)	Units acquired
1	50.00	6.000
2	40.00	7.500
3	32.00	9.375
4	40.00	7.500
5	50.00	6.000
6	60.00	5.000
Total / average	45.33	41.375

Source: own calculations; hypothetical example before taxes and costs.

The average purchase price is 1,800/41.375 = 43.50 euro and lies below the arithmetic average price of 45.33 euro. The interpretation, however, calls for sobriety: Constantinides (1979) shows that instalment investing is inferior in expectation to immediate lump-sum investment when capital is already available; Statman (1995) explains its popularity on behavioural grounds. For the typical saver, whose capital only arises with income, the investment plan is nevertheless the natural vehicle — and compound interest weights its instalments: the early instalments work longest and contribute most to terminal wealth, which once again makes starting early the most important single decision.

The size of the purchase price advantage depends on volatility: the difference between the arithmetic and the harmonic mean grows approximately in proportion to the variance of the purchase prices — in calm markets the effect is small, in volatile ones tangible. This changes nothing about its classification as a discipline rather than a return instrument, but explains why the savings plan plays out its behaviourally stabilising strength precisely in turbulent phases: it mechanically translates price declines into higher unit counts and removes the basis for stress-driven decisions.

The normative assessment should also name the alternative: for the investor with capital in hand, the relevant choice is not savings plan versus nothing but staggered versus immediate entry — and here expected value favours the immediate one, because the capital is invested longer on average (Constantinides, 1979). For the investor without initial capital the question does not arise: his savings plan is not a timing decision but the only possible mapping of his income path into wealth. The widespread conflation of the two situations explains a considerable part of the controversy surrounding the cost-average effect.

[14] Dichev (2007) shows that dollar-weighted investor returns have historically lagged time-weighted index returns — timing behaviour costs return.

7 Growth under Uncertainty: Volatility Drag and Horizon Effects

7.1 Arithmetic and Geometric Mean: The Volatility Drag

Under uncertainty, capital does not grow at the average (arithmetic) return μ but at the geometric rate g. The canonical example: +50 percent is followed by −50 percent; the arithmetic mean is zero, yet 1 euro turns into $1.50 \cdot 0.50 = 0.75$ euro. For lognormal returns, approximately $g \approx \mu - \sigma^2/2$ holds: half the variance is withdrawn from growth (volatility drag). Because compound interest acts multiplicatively, it punishes fluctuations — losses require disproportionate gains to be recovered.[15]

The same logic underpins growth-optimal portfolio theory: Kelly (1956) and Latané (1959) show that maximising the expected logarithmic return $E[\ln(1+R)]$ maximises long-run capital growth; in Merton's (1969) continuous-time model, the growth-optimal share of risky assets is $f^* = (\mu - r_f)/\sigma^2$, hence directly a function of the ratio of excess return to variance.

The distinction between mean and median is essential: for lognormally distributed terminal wealth, the expected value lies above the median owing to the right skewness of the distribution and is attained by only a minority of paths — the typical (median) investor experiences growth at the rate g, not at μ. Projections based on arithmetic average returns therefore systematically overstate probable terminal wealth, and the gap widens with variance and horizon. The drag, finally, has a constructive flip side:

$$R_{div} \approx (\sigma^2 - \sigma_P^2) / 2 \tag{16}$$

Equation (16) is the diversification or rebalancing return (Fernholz and Shay, 1982; Booth and Fama, 1992): the geometric return of a regularly rebalanced portfolio exceeds the weighted average of the geometric single-asset returns by approximately half the difference between the average single-asset variance σ^2 and the portfolio variance σ_P^2. With single stocks at 25 and a portfolio at 15 percent volatility, this amounts to roughly 2 percentage points per year — diversification raises the geometric growth rate without changing the expected single-asset return. Table 6 quantifies the volatility drag for an arithmetic return of 7 percent over 20 years.

[15] The term variance drain is due to Messmore (1995). For lognormally distributed terminal wealth, the median equals $\exp(T \cdot (\ln(1+\mu) - \sigma^2/2))$; the expected value lies above it but is attained by only a few paths.

Table 6: Volatility drag over 20 years (initial capital 100,000 euro, arithmetic return 7 percent p.a.)

Volatility σ	$g \approx \mu - \sigma^2/2$	Terminal wealth $(1+g)^{20}$ (€)	Median lognormal (€)
0%	7.00%	386,968.45	386,968.45
10%	6.50%	352,364.51	350,143.53
20%	5.00%	265,329.77	259,392.71
30%	2.50%	163,861.64	157,329.63

Source: own calculations; hypothetical example before taxes and costs.

At 20 percent volatility, the drag costs more than 120,000 euro over 20 years relative to the fluctuation-free path — volatility is thus a direct cost factor of compound interest. This yields an often overlooked rationale for diversification: lowering volatility at a given μ raises the geometric growth rate g and hence attainable terminal wealth. Disciplined rebalancing works in the same direction by controlling portfolio variance.

At the single-stock level the drag is even more drastic: because individual equities exhibit volatilities of 30 percent and more, their geometric return is frequently negative despite a positive arithmetic expectation — historically, the majority of US stocks lagged Treasury bills while a few extreme winners carried the index return (Bessembinder, 2018). The index portfolio harvests the skewness of this distribution, the concentrated portfolio bets against it; the volatility drag is thus the quantitative argument for understanding diversification not as a sacrifice of return but as a condition of growth.

7.2 Horizon Effects: The Convexity of Time

The wealth multiplier $(1+g)^T$ is convex in the horizon T: equal spans of time at the end of an investing life generate larger absolute gains than at the beginning. Table 7 shows multipliers and doubling times for growth rates between 2 and 8 percent.[16]

Table 7: Wealth multipliers $(1+g)^T$ and doubling times

Growth rate g	10 years	20 years	30 years	40 years	Doubling time (years)
2%	1.22	1.49	1.81	2.21	35.0
4%	1.48	2.19	3.24	4.80	17.7
6%	1.79	3.21	5.74	10.29	11.9
8%	2.16	4.66	10.06	21.72	9.0

Source: own calculations; hypothetical example before taxes and costs.

At 6 percent, capital increases nearly tenfold over 40 years; a good half of that tenfold increase falls on the final ten years. Starting ten years later therefore costs, economically, not the first but the last and most profitable decade. At the same time, caution is warranted: the horizon does not substitute for risk-bearing capacity, since the dispersion

[16] Samuelson (1963) objects to so-called time diversification on the grounds that risks do not offset over time: the dispersion of terminal wealth grows with the horizon.

of possible terminal wealth also grows with time (Samuelson, 1963). The practical lesson of horizon effects remains untouched by this: starting early dominates any market timing as a decision variable, because it opens up the most convex stretch of the compounding curve.

The international long-run evidence supports the horizon perspective: in the century-long accounting of Dimson, Marsh, and Staunton (2002), equities earned higher real returns than bonds and money market investments in all 16 countries examined — with substantial variation over time and marked differences across countries. At the same time, the authors caution against extrapolating historical returns unexamined: the time lever acts on the realised, not the hoped-for, growth rate.

Stated precisely, the horizon changes the shape of risk, not its existence: as T grows, the probability of falling behind the safe asset declines, but the magnitude of possible shortfalls grows — which is why the price of insuring terminal wealth rises rather than falls with the horizon (Bodie, 1995). The long breath is thus no licence for arbitrary risk, but the precondition for the positive growth rate g to unfold its convex effect; risk-bearing capacity and horizon are complementary, not substitutable, resources.

8 The Psychology and Biology of Compound Interest: The Delay Effect

8.1 Exponential Growth Bias: Evidence and Modelling

Compound interest appears inconspicuous at first — the curve runs flat — and unfolds most of its effect only in later years. It is precisely this shape that collides with human intuition. Cognitive psychology has documented since the 1970s that people notoriously underestimate exponential growth and instead extrapolate linearly: Wagenaar and Sagaria (1975) show that intuitive extrapolations of exponential series come out dramatically too low, and Wagenaar and Timmers (1979) demonstrate that additional data points paradoxically do not improve the estimates. Christandl and Fetchenhauer (2009) find the bias among experts as well — it is thus not an educational but a perceptual phenomenon.[17]

Formally, following Levy and Tasoff (2016), the bias can be represented as a convex combination of linearly perceived and actual exponential growth: $\tilde{V}(i, T) = K0 \cdot [(1 - \alpha) \cdot (1 + i \cdot T) + \alpha \cdot (1 + i)^T]$ with $\alpha \in [0, 1]$, where $\alpha = 1$ corresponds to correct compound arithmetic and $\alpha = 0$ to fully linear thinking. The perception error grows with the horizon: at 7 percent over 30 years, the true terminal value is 7.6 times the stake, the linearly estimated one only 3.1 times — the fully biased investor underestimates his terminal wealth by roughly 59 percent, and the underestimation grows with every additional year.

[17] In the experiments of Wagenaar and Sagaria (1975), the intuitive estimates of up to two thirds of subjects fell below 10 percent of the correct value.

The economic consequences are substantial. Stango and Zinman (2009) show that more biased households save less and borrow at higher cost. Levy and Tasoff (2016) estimate roughly one third of a representative US sample to be fully biased and quantify the move from full bias to accurate perception at, ceteris paribus, 55 to 90 percent higher accumulated assets. Almenberg and Gerdes (2012) link the bias to low financial literacy; Foltice and Langer (2018) document it even among students with financial training. The flat beginning of the compounding curve is therefore not merely an optical but a behaviourally relevant problem: it withholds the reward from the saver precisely in the phase in which the saving decision would need to be reinforced — the delay effect of compound interest.

The bias is remarkably robust to expertise and can at the same time be addressed deliberately: McKenzie and Liersch (2011) show that even MBA students extrapolate the growth of retirement wealth largely linearly — but that graphical depiction of the exponential path significantly raises saving intentions. A concrete design recommendation for investor communication follows: terminal wealth calculators and growth charts are not marketing instruments but corrective lenses for a documented perceptual error.

How consequential linear thinking about exponential processes is was demonstrated most recently by the COVID-19 pandemic: Lammers, Crusius, and Gast (2020) show that people massively underestimated the growth of case numbers and that correcting this misperception increased support for protective measures. The finding is structurally identical to the savings problem — the same cognition, the same underestimation, the same correctability — and demonstrates that exponential growth bias is not a peculiarity of the financial domain but a general property of human intuition.

8.2 Present Bias, Neuroeconomics, and Commitment

A second, motivational hurdle joins the misperception of growth: present bias. In Laibson's (1997) quasi-hyperbolic discounting, a decision-maker values the utility stream according to $U_t = u(c_t) + \beta \cdot \Sigma\, \delta^k \cdot u(c_{t+k})$ with $\beta < 1$: all future periods are additionally devalued relative to the present, which generates dynamic inconsistency — the savings plan scheduled for tomorrow is postponed again today (Frederick, Loewenstein, and O'Donoghue, 2002). Compound interest demands precisely the patience that this preference structure systematically undermines: its reward lies in the distant, heavily discounted future, while its cost — forgone consumption — lies in the present.[18]

Neuroeconomics provides the biological foundation: McClure, Laibson, Loewenstein, and Cohen (2004) show that immediately available rewards activate the evolutionarily old limbic system, whereas weighing delayed rewards falls to the younger prefrontal cortex. In Kahneman's (2011) terminology, compound interest confronts the fast, intuitive System 1

[18] Using functional magnetic resonance imaging, McClure et al. (2004) show that immediate rewards primarily activate limbic structures, whereas delayed rewards are associated with prefrontal and parietal activity.

with a task that requires the slow, calculating System 2. This is evolutionarily plausible: in the environment in which the human brain evolved, hardly anything grew exponentially over decades — linear heuristics were sufficient, and there was no selection pressure for rewards forty years away.

The finding implies the response: where perception and self-discipline fail systematically, automatisms must step in. Thaler and Benartzi (2004) show with the Save More Tomorrow programme that coupling automatic savings increases to future pay rises lifted participants' saving rates from 3.5 to 13.6 percent within 40 months. The automated savings plan, the accumulating share class, and the standing order are, economically, nothing but commitment technologies that protect compound interest from one's own psychology.

A third behavioural hurdle concerns perseverance: Benartzi and Thaler (1995) explain the historically high equity risk premium by myopic loss aversion — the combination of narrow framing and frequent portfolio evaluation. Checking the portfolio daily produces losses almost as often as gains, weighted about twice as heavily under prospect theory; infrequent checking shows predominantly gains. For compound interest, the evaluation frequency is thus a behavioural variable of the first order: long evaluation intervals support precisely the holding periods that Section 10 identifies as decisive.

The most effective commitment, finally, operates before the decision: Madrian and Shea (2001) show that merely changing the default — automatic enrolment in the retirement plan with a right to opt out instead of active sign-up — lifted the participation rate of new employees from roughly one half to over 85 percent. Inertia, which otherwise prevents the start of saving, works for compound interest under changed defaults: the architecture of the decision replaces the willpower of the decision-maker.

9 The Time Factor Beats the Interest Rate: The Mathematics of the Savings Plan

9.1 Terminal Value Formulas and Sensitivity Analysis

For a lump sum, $FV = K0 \cdot (1 + i)^T$; for a savings plan with end-of-period instalment R over n periods at periodic rate i, the terminal value follows the future value formula of an annuity, $FV = R \cdot ((1 + i)^n - 1) / i$. The levers of both formulas can be compared precisely via elasticities: for the lump sum, $\partial \ln FV / \partial \ln(1 + i) = T$ and $\partial \ln FV / \partial \ln T = T \cdot \ln(1 + i)$ — both sensitivities grow linearly in the horizon T. Time is thus the only parameter that multiplies the effect of all others: a longer horizon raises not only the exponent but also the sensitivity of terminal wealth to every single point of return.[19]

[19] Calculations assume end-of-month instalments and an effective annual rate of 5 percent; the equivalent monthly rate is $i_m = 1.05^{1/12} - 1 = 0.4074$ percent.

A numerical comparison makes the ranking tangible: 10,000 euro grows at 5 percent over 20 years to 26,533 euro. Doubling the interest rate to 10 percent yields 67,275 euro; doubling the horizon to 40 years instead yields 70,400 euro. Doubling time beats doubling the interest rate — and, unlike the return, time is a variable the investor controls. The ranking of the three levers of the savings plan follows the same logic: doubling the instalment exactly doubles terminal wealth (the instalment enters linearly), doubling the interest rate acts more than proportionally through the exponent, and doubling the horizon dominates both at usual parameters — with the additional virtue of requiring neither forgone consumption nor higher risk, merely an earlier start.

$$FV = R \cdot [(1 + i)^n - (1 + g)^n] / (i - g) \tag{17}$$

Equation (17) is the future value of a growing annuity: the savings plan whose instalment R rises at rate g per period — for instance in step with wage growth or inflation. It connects savings-plan mathematics with the commitment strategy of automatic contribution increases (Thaler and Benartzi, 2004) and keeps the real saving effort constant where a fixed nominal instalment would lose purchasing power. Table 8 shows how even small constant monthly instalments grow into large fortunes over decades.

Table 8: Terminal values of monthly savings instalments at a 5 percent effective annual rate

Monthly instalment (€)	**10 years (€)**	**20 years (€)**	**30 years (€)**	**40 years (€)**
100	15,436.32	40,580.45	81,537.59	148,252.46
200	30,872.63	81,160.90	163,075.18	296,504.92
300	46,308.95	121,741.35	244,612.77	444,757.38
Contributions (per EUR 100 instalment)	12,000.00	24,000.00	36,000.00	48,000.00

Source: own calculations; hypothetical example before taxes and costs.

With a 100 euro monthly instalment, 148,252 euro after 40 years stands against a contribution total of only 48,000 euro: a good two thirds of terminal wealth are income and income on income. The non-linearity in time is striking: the fourth decade alone contributes 66,715 euro — almost as much as the first three decades combined (81,538 euro), although only a quarter of the instalments flows in it. Terminating the plan early therefore removes not an arbitrary but the most profitable stretch of the curve.

For completeness, the time lever must be read in real terms: by the Fisher equation (10), real wealth grows at the real rate. At 6 percent nominal and 2 percent inflation, capital multiplies 10.3-fold over 40 years in nominal terms, but only 4.66-fold in real terms — still impressive, yet a different order of magnitude. Savings goals formulated in today's purchasing power must therefore be computed with real growth rates; the power of compound interest must be earned in real terms before it can be admired in nominal ones.

9.2 The Early Start: A Two-Saver Experiment

How heavily the early years weigh is shown by a classic thought experiment. Saver A invests 200 euro per month from age 25 to 35 and then stops contributing entirely; saver B starts at 35 and saves the same 200 euro per month for thirty years until 65. A contributes 24,000 euro; B, at 72,000 euro, three times as much. Table 9 shows the result at a 6 percent annual return.[20]

Table 9: Early versus late start of saving (monthly instalment 200 euro, 6 percent effective annual rate)

Position	Saver A (25 to 35)	Saver B (35 to 65)
Total contributions	24,000.00	72,000.00
Portfolio value at 35	32,494.69	—
Terminal wealth at 65	186,632.96	194,902.59
Terminal wealth per euro contributed	7.78	2.71

Source: own calculations; hypothetical example before taxes and costs.

With one third of the contributions, A reaches 96 percent of B's terminal wealth: every euro contributed by saver A grows to 7.78 euro, every euro of saver B only to 2.71 euro. The ten early years almost fully replace thirty late ones, because the 32,495 euro of capital in place at age 35 has three decades of uninterrupted compounding ahead of it. Small amounts early beat large amounts late — the most important return of an investing life is the decision to start.

The experiment connects with the commitment logic of Section 8: escalating the savings rate automatically with income (Thaler and Benartzi, 2004) combines the time effect of the early years with a rising instalment path and at the same time neutralises the present bias that most frequently prevents the early start in practice.

10 Time in the Market Beats Market Timing

10.1 The Arithmetic of Timing

If time is the strongest lever of compound interest, its greatest enemy is interruption. Market timing — the attempt to avoid downturns and capture upswings — intervenes in the chaining of growth factors $FV = K0 \cdot \Pi\ (1 + r_t)$: absence from the market in period t replaces the factor $(1 + r_t)$ with 1. Because equity returns accrue in an extremely concentrated fashion, the expected value of this intervention is negative. Sharpe (1975) shows, already for annual data of the S&P Composite 1934 to 1972, that timing only pays

[20] The example assumes a 6 percent effective annual return and end-of-month instalments; saver A lets her capital grow without further contributions from age 35.

beyond a forecasting accuracy of roughly 74 percent — a threshold that hardly any forecaster reaches persistently.[21]

Estrada (2008) sharpens the finding at the daily level: across 15 developed equity markets, missing only the ten best trading days would have reduced terminal wealth by 50.8 percent on average; for the Dow Jones 1900 to 2006, the loss amounts to 65 percent. Moreover, the best days frequently follow immediately after the worst, so that exiting in a downturn almost inevitably misses the recovery. The distribution of daily returns is heavily fat-tailed — a few outliers, black swans in the sense of Taleb (2007), carry the compounding of the entire market.

Theoretically, the scepticism is grounded in the efficient markets hypothesis: if prices are informationally efficient, they approximately follow a martingale, and systematically profitable timing signals should not exist (Fama, 1970). The timing question is thus not a question of diligence but of the market's own information processing; the evidence that follows shows that this theoretical presumption is borne out in observed investor behaviour.

For completeness, the literature does document limited predictability of long-horizon returns: valuation ratios such as the dividend-price ratio explain part of the variation in returns over horizons of years to decades (Campbell and Shiller, 1988). For timing at horizons of days to months, however, little follows — the predictive power is too weak and too slow to reach Sharpe's (1975) threshold, and exploiting it requires long, uninterrupted holding periods of its own.

For the savings plan investor, the timing question arises in milder form — as the question of when to invest the next instalment. The arithmetic of interruption applies here too: suspending instalments to wait for lower prices trades a certain gain of time for an uncertain saving in price, and the concentration of the best trading days makes this trade loss-making in expectation. The automatic, level-independent entry of the savings plan is thus not merely a disciplining device (Section 6.2) but the consistent answer to the timing evidence of this section.

10.2 Empirical Evidence on Investor Behaviour

The behavioural evidence confirms the arithmetic. Barber and Odean (2000) show that the most active retail investors earned 11.4 percent per year net of costs while the market returned 17.9 percent — trading activity is, in the aggregate, harmful to returns. Dichev (2007) measures dollar-weighted investor returns, which incorporate the effect of chosen entry and exit dates, and finds them roughly 1.3 percentage points per year below time-weighted buy-and-hold returns on the NYSE and AMEX. Jeffrey (1984) stresses the asymmetry of timing: the upside of the perfect exit is bounded, the downside of the missed upswing is not. Bessembinder (2018), finally, shows that only about 4 percent of all US

[21] In Estrada (2008), the ten best days correspond to less than 0.1 percent of all trading days considered.

stocks generated the entire net wealth creation over Treasury bills since 1926 — returns are extremely concentrated not only across days but also across stocks, so that selective and sporadic investing misses the few carriers of compounding with high probability.[22]

The pattern is confirmed at the fund level as well: Friesen and Sapp (2007) put the timing costs of flow movements in US equity funds at roughly 1.6 percentage points per year — investors buy after price rises and sell after declines, so that their dollar-weighted return systematically falls short of the fund return. Table 10 summarises the evidence.

Table 10: Empirical evidence on market timing and trading activity

Study	Market and period	Central finding
Sharpe (1975)	USA, S&P Composite 1934-1972	Timing only pays beyond ca. 74% forecasting accuracy
Jeffrey (1984)	USA, S&P 500 1926-1982	Asymmetry: missed upswings weigh more than avoided downturns
Barber and Odean (2000)	USA, 66,465 accounts 1991-1996	Most active traders 11.4% p.a. vs. 17.9% market return
Dichev (2007)	NYSE/AMEX 1926-2002	Dollar-weighted returns ca. 1.3 pp p.a. below buy-and-hold
Estrada (2008)	15 developed markets; Dow 1900-2006	Missing 10 best days: −50.8% (Dow: −65%) terminal wealth
Bessembinder (2018)	USA, all stocks 1926-2016	4% of stocks generate the entire net wealth creation

Source: compiled from the studies cited.

For the retail investor, this implies no capitulation to the market but a division of labour: the market delivers the return, time delivers the power, and the investor's task is to let both work undisturbed. Time in the market replaces the unanswerable question of the right moment with the answerable question of the right holding period — and thereby joins the psychology of Section 8, the savings-plan mathematics of Section 9, and the timing evidence of this section into a single rule of action: start early, continue automatically, stay invested.

For portfolio construction this does not render every active decision obsolete — but it does mean that the capital commitment itself should not be up for debate. A broadly diversified, low-cost core that remains permanently invested secures participation in the few days and stocks that carry the market's compounding; tactical positions are confined to a satellite share whose failures cannot interrupt the core wealth. Thus the investor's ambition is preserved — and the power of compound interest left untouched.

[22] Barber and Odean (2000) examine 66,465 household accounts at a large US discount broker over the years 1991 to 1996.

11 Conclusion and Recommendations

The power of compound interest is the product of three multiplicatively linked quantities: return, time, and the reinvestment ratio. The worked examples put figures on their contributions — forgoing the reinvestment of distributions costs 21.1 percent of total wealth in the 20-year example; for the ten-year coupon bond, up to a quarter of interest income stems from interest on interest, depending on the reinvestment rate; and volatility withdraws a directly quantifiable share from growth according to $g \approx \mu - \sigma^2/2$. Day-count conventions shift interest amounts by up to 1.4 percent per period and make the effective annual rate the only reliable basis of comparison. Behavioural economics explains at the same time why this power so often remains unexploited: exponential growth bias makes the flat beginning of the curve appear disappointing, present bias postpones the start of saving, and market timing interrupts the chaining of growth factors at its most valuable points — in the two-saver experiment, the early starter reaches 96 percent of the late starter's terminal wealth with one third of the contributions.[23]

For averagely sophisticated retail investors, the findings condense into seven recommendations: first, start early, because the convexity of time makes the final years of the horizon the most profitable and the time factor dominates the interest rate as a lever; second, reinvest income consistently — accumulating vehicles automate this; third, minimise costs and taxes, since they turn the same compounding mechanism against the investor; fourth, when comparing interest offers, always look at effective annual rates on a uniform convention; fifth, control volatility through diversification and rebalancing, because doing so directly raises the geometric growth rate; sixth, automate saving in order to bypass exponential growth bias and present bias; seventh, stay invested, because time in the market beats market timing.

Beyond the individual level, the findings have an educational policy dimension: basic financial literacy — in particular an understanding of compound interest — is among the most robust predictors of retirement planning and wealth accumulation (Lusardi and Mitchell, 2014). A paper that measures the power of compound interest is in this respect also an argument for making that power a subject of school curricula and investor communication: the return on financial education is — like the capital it acts upon — exponential.

The same mechanism counsels cost discipline: recurring fees are negative compound interest. One percentage point of annual costs reduces terminal wealth over 30 years by roughly a quarter at a 7 percent gross return — not because one percent were much, but because it is withdrawn from the growth base in every single period and thereby forfeits all the future income it would have earned on itself. A product's expense ratio is therefore

[23] This paper serves informational and educational purposes only; it does not constitute an investment recommendation.

no footnote but, alongside savings rate, return, and time, the fourth parameter of the terminal value equation.

The limitations of the analysis lie in its simplifications: constant returns, no taxes or costs, hypothetical examples. Future work could extend the results with stochastic simulations of the terminal wealth distribution, the impact of country-specific taxation, and the decumulation phase, in which compound interest acts on withdrawal plans with the opposite sign (sequence-of-returns risk).

In closing, the image in the title deserves precision: the power of compound interest is not a force of nature that befalls the investor but one that he harnesses or squanders. It demands no forecasting ability, no superior knowledge, and no talent — only an early start, an automated process, and the willingness not to interrupt it for decades. That precisely these three requirements are the hardest for human psychology is the true point of this paper — and the reason why understanding it is itself an investment with an exponential return.

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